Torsional Displacement for Asymmetric Low-Rise Buildings with RC C-shaped Cores

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ABSTRACT: C-shaped reinforced concrete (RC) walls (or cores) are a popular choice in design and construction that are commonly used in Australian practice as the lateral load resisting elements in a building. Many low-rise buildings incorporate C-shaped walls close to the perimeter of the building to make efficient use of the floor area. Due to these types of structural elements being inherently stiffer than other elements, the building is asymmetric in plan. Consequentially, this can produce a large increase in the displacement demand on other structural elements in the building due to torsional response in the event of an earthquake. This study investigates the increase in peak displacement demand of a low-rise asymmetric building that incorporates C-shaped cores. The RC walls are initially designed to the current Australian Standards for a 500-year return period earthquake in Melbourne. The displacements of the flexible edge of buildings are calculated using time-history analyses for displacement response of a single-degree-of-freedom structure with torsionally coupled vibration modes. The displacement demands of the building are verified using a dynamic time-history analysis in a finite element modelling program. A simplified expression is given to approximate the torsional displacement, which could be used in a displacement-based assessment.

1 INTRODUCTION

The channel-shaped (C-shaped) wall or core, sometimes referred to as U-shaped, is one of the most geometrically popular reinforced concrete (RC) structural walls used in practice (Beyer, 2007). Due to the increasing demand for more efficient use of the building area, particularly warranted by architects, the C-shaped core is sometimes placed on the perimeter of the building as shown in Figure 1. The C-shaped core is typically much stiffer than other structural elements in the building which creates asymmetry in plan in terms of the distribution of stiffness and strength. Thus, this type of building configuration has the potential to create large torsional response in the event of seismic ground motions. Structural damage due to torsional effects has been observed and reported from past earthquake events (Esteva, 1987; Gokdemir *et al.*, 2013; Hart, 1975). The Christchurch earthquake in 2011 also provided some more recent observations of damage that was most likely caused by a 'torsionally sensitive response' (Dizhur *et al.*, 2011). Importantly, structural and non-structural elements can be severely damaged in areas of the building where the maximum torsional response is produced (e.g. the "flexible edge"). This could result in the catastrophic collapse of the building. Moreover, in regions of low-to-moderate seismicity, such as Australia, most of the existing buildings have not been designed for earthquake loading and the corresponding RC structural elements sometimes have very limited ductility (Hoult *et al.*, 2014; Lumantarna, 2012; Wilson *et al.*, 2015).

This paper follows on from research conducted by Lumantarna *et al.* (2013) and is focused on investigating the peak displacement demand (PDD) of asymmetric low-rise RC buildings that incorporate C-shaped cores. A parametric study is initially undertaken to investigate the parameters that PDD is dependent upon. Two case study buildings with different C-shaped core configurations are investigated to determine their performance and the PDD caused by a torsional response. A simplified equation is derived using the results from the initial parametric study to calculate an estimated PDD for buildings that incorporate similar C-shaped wall configurations, which can be used for design and assessment purposes.
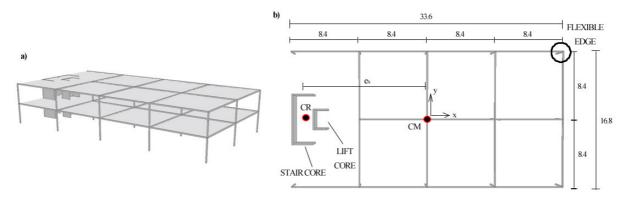
Figure 1. (a) Isometric and (b) plan view of building type 1 with two C-shaped walls

2 METHODOLOGY

Torsional actions in buildings and the displacement response of such phenomenon have been the subject of much research in the past (Chandler & Hutchinson, 1988; Goel & Chopra, 1991). However, the study of the seismic performance of asymmetric RC buildings that incorporate just one major wall (or core) are very limited as 'this particular structural configuration is usually found in regions of low seismicity' (Peng & Wong, 2008). Although the research by Beyer (2007) focused on the seismic design of torsionally eccentric buildings with U-shaped walls, the overall study of the torsional response was based on buildings with rectangular RC walls. This was ultimately because it was thought that the 'U-shaped wall can be idealised as two rectangular walls since in most cases the torsional stiffness of the U-shaped wall is relatively small compared with the rotational stiffness of the entire structure' (Beyer, 2007). However, it is argued by Peng and Wong (2008) that the torsional stiffness of the major wall in a single-wall-frame building might be significant.

Research by Lumantarna *et al.* (2013) examined asymmetric buildings that are controlled mainly by the displacement demand properties of the base excitation (or RSD_{max}), which is thought to control the response of buildings in intraplate regions of low-to-moderate seismicity where mild ground shaking is to be expected. The work represented a new development in assessing the performance of asymmetric buildings in low-to-moderate seismic regions, such as Australia, by incorporating the displacement controlled phenomenon into the torsional action analysis. The results from the study showed that the PDD value was constrained 'within 1.6 times the value of the maximum elastic response spectral displacement (RSD_{max})' (Lumantarna *et al.*, 2013).

Using the equations of dynamic equilibrium, Lumantarna *et al.* (2013) have calculated the modal coefficients of the natural periods of vibration for the elastic torsional coupling behaviour of a single-storey building. This is incorporated in a time-history analysis for displacement response of a single-degree-of-freedom (SDOF) structure to ultimately give the displacement response of the building.

The first and second natural angular velocity ratios (λ_1 and λ_2 respectively) are calculated as:

$$\lambda_{1,2} = \frac{1 + (b_r^2 + e_r^2)}{2} \pm \sqrt{\left(\frac{1 - (b_r^2 + e_r^2)}{2}\right)^2 + e_r^2} \qquad (1)$$

where e_r and b_r are found from Equations 2, 3 and 4.

$$e_r = \frac{e_x}{R} \qquad (2)$$

$$b_r = \sqrt{\frac{K_t}{K_x}} \cdot \frac{1}{R} \qquad (3)$$

$$R = \sqrt{\frac{(2B)^2 + D^2}{12}} \qquad (4)$$

R is the mass radius of gyration and B and D are the dimensions of the building (as shown in Figure 2). The translational stiffness of the building about the x and y axis (K_x and K_y), which is primarily from the stiffness of the structural walls, can be found from an initial pushover analysis. The torsional stiffness of the building (K_t) is found by using Equation 5, where x_i and y_i represent the location of wall i.

$$K_t = \sum_{i=1}^{n} K_x(x_i - e_x)^2 + \sum_{i=1}^{n} K_y(y_i - e_y)^2 \qquad (5)$$

The eccentricity in the x direction (e_x), can be calculated using Equation 6.

$$e_x = \frac{\sum_{i=1}^{n} K_{xi} x_i}{\sum_{i=1}^{n} K_{xi}} \qquad (6)$$

The angular velocities of the coupled modes of vibration are then calculated (for ground motions parallel to the y-axis in Figure 1):

$$\Omega_{1,2} = \lambda_{1,2}\omega_x = \lambda_{1,2}\sqrt{\frac{K_x}{m_e}} \qquad (7)$$

The participation factors (PF) to be included in the time-history analysis are found using Equation 8.

$$PF_{1,2} = \frac{1}{1 + \left(\dfrac{\lambda_{1,2}^2 - 1}{e_s}\right)^2} \qquad (8)$$

The displacement response of the "flexible" (x_{+B}) and "stiff" (x_{-B}) edges of the building are thus calculated using Equation 9.

$$x \pm_B = \sum_{i=1}^{2} (1 + (\lambda_i^2 - 1)(\frac{\pm B}{e}))PF_i \times U_{\Omega_i, \zeta(t)} \qquad (9)$$

where $U_{\Omega_i, \zeta(t)}$ is the time-history displacement response of a SDOF representation for Ω_i of the asymmetric structure, solved using the central difference method (Chopra, 2011):

$$u_{j+1} = \frac{-\ddot{u}_{gJ} + Au_j + Bu_{j-1}}{C} \qquad (10)$$

$$A = -\omega_n^2 + \frac{2}{\Delta t^2} \qquad (11)$$

$$B = \frac{\xi \omega_n}{\Delta t} \qquad (12)$$

$$C = \frac{1}{\Delta t^2} + \frac{\xi \omega_n}{\Delta t} \qquad (13)$$

where Δt is the time step of the displacement-time history, ξ is the damping ratio, u_j is the displacement response and $\ddot{u}_g$ is the ground acceleration.

Lumantarna *et al.* (2013) has previously investigated the torsional response of buildings with rectangular walls in both principle directions that are based on a single-storey building model. Furthermore, the values for the parameters of e_r and b_r were within a range of 0.1-0.8 and 0.4-1.4 respectively. This is extended here to consider buildings that have larger eccentricities and are more torsionally flexible. The possible assistance of any frames in the building has not been considered

here and will be the subject of a future study.

Using Equations 1-13, it is possible to investigate a range of parameters for typical low-rise structures to determine how sensitive the PDD of the flexible edge is to these parameters. The parametric study aims to find obvious trends as well as the most sensitive parameters to use in a case study.
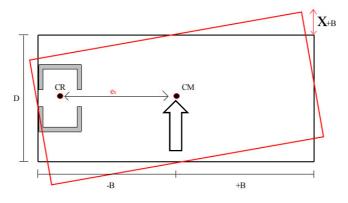

Figure 2. Plan view of building type 2 with two C-shaped walls

3 PARAMETRIC STUDY

The parametric study focused on two building configurations, which differ by the direction and geometry of the walls as shown in Figure 1 (Type 1) and Figure 2 (Type 2). Building Type 1 consists of two C-shaped walls; an elevator core that encloses two cars, which are required to carry 2x500kg (6 person) as per the recommendations from RLB (2014), and a stair shaft to enclose 1250mm wide flights and landings. Building Type 2 also consists of two C-shaped walls of same dimensions; a stair shaft similar to Type 1 and an elevator core which encloses 3x900kg (12 person) cars. The dimensions of the walls are given in Table 1. The range of parameters used in the study are summarised in Table 2, the values chosen based on realistic buildings.

Table 1. Dimensions of the C-shaped walls used in building types 1 and 2

Building Type	Core	Web (mm)	Flange (mm)	Thickness (mm)
1	Stair	6300	2650	200
1	Elevator	2600	1700	200
2	Stair	6300	2650	250
2	Elevator	6300	2650	250

Table 2. Parameters used in parametric study

Parameter	Range
Number of Storeys	1 - 5
Dead Load (kPa)	4 - 8
Live Load (kPa)	1 - 4
Floor Area (m^2)	360, 540, 564.5, 810, 1350
2B (m) x D (m)	18x20, 27x20, 27x30, 33.6x16.8, 45x30
e_x (m)	8.1 - 21.8
b_r	0.035 - 0.11
e_r	1.05 – 1.49
Longitudinal Reinforcement Ratio (%)	0.15-1.00

SeismoArtif (SeismoSoft, 2013a) was employed in creating ten artificial ground motions for two different soil classes that would be used in a time-history analysis to assess the PDD of the asymmetric

buildings. The results of a Probabilistic Seismic Hazard Analysis (PSHA) using the AUS5 recurrence model (Brown & Gibson, 2004) for a 500-year return period in Melbourne were used as the target spectra for the artificial ground motions soil class B. SHAKE-2000 (Ordonez, 2013) was used to create the response at the ground level of a soil class C for a 500-year return period event for Melbourne. The target spectra and resulting displacement response spectra of the artificial ground motions are shown in Figure 3(a) and (b) for soil class B and C respectively.

The parametric study is carried out using Equations 1-13 for each artificial ground motion. For each different building (represented by the different floor area), the number of storeys is held constant while the weight of the building is incrementally increased (with a different G and Q combination). The average PDD of the 10 ground motions used for each building type is then calculated. The same process is then undertaken for the buildings with a different number of storeys, within a range of 1-5 to represent a typical range for low-rise buildings.
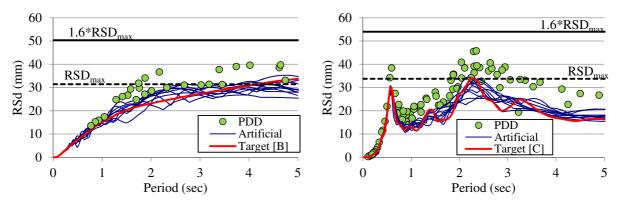

Figure 3. Displacement response of artificial motions for (a) soil class B, building Type 1 and (b) soil class C, building Type 2
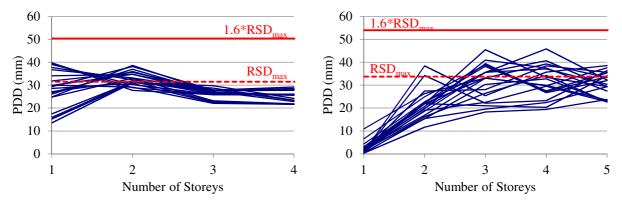

Figure 4. PDD as a function of the number of storeys for different floor areas and for (a) building Type 1 and (b) building Type 2

The parametric study results for Building Type 1, using the artificial motions for site class B, gave an increasing PDD on the flexible edge of the building as the ratio of width to depth of the building (2B/D) increased. The largest PDD of the flexible side for this building type corresponded with a one or two storey building, illustrated in Figure 4(a). In contrast, the results for building Type 2, using the artificial motions for site class C, indicated that the PDD increases with the number of storeys with maximum displacement at 3 or 4 storeys. These results are illustrated in Figure 4(b). Type 2 also gave results which tended to increase the PDD with an increasing 2B/D ratio, although this relationship was not as prominent as it was in the results for Type 1. What is apparent from the results for both building types is that the PDD of the flexible edge of the buildings (critical side) are controlled by the 1st torsional coupled modal natural period (T_{1m}). The results of the PDD and corresponding T_{1m} of the buildings have been superimposed in Figures 3(a) and (b) for Type 1 and 2 respectively. The largest PDD occurs for when buildings have a T_{1m} coinciding with the period at or close to RSD_{max}. All of the results of PDD are also constrained within 1.6 x RSD_{max}, which conforms with findings from Lumantarna *et al.* (2013). The RSD_{max} (average of the maximum displacement

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response of the corresponding 10 artificial ground motions) and 1.6 x RSD_{max} have also been indicated in Figure 3. These findings can be used to derive a simplified expression to estimate the PDD of the flexible side of the building for C-shaped walls that are placed in a similar configuration to building types 1 and 2.

4 ESTIMATION OF PDD

This simplified expression requires fewer parameters to be known compared to the previous time-history analysis method, and is also a less rigorous and time consuming method. The method, which can be applied to a displacement-based assessment for asymmetric buildings, firstly calculates the 1st torsional modal natural period (T_{1m}) of the building (Equation 14):

$$T_{1M} = \frac{2\pi}{\lambda_1 \omega_x} = \cfrac{2\pi}{\sqrt{\frac{K_x}{m_e}} \cdot \sqrt{\frac{1 + (e_r^2 + b_r^2)}{2} - \sqrt{\left(\frac{1 - (e_r^2 + b_r^2)}{2}\right)^2 + e_r^2}}} \tag{14}$$

Since the PDD does not exceed 1.6 x RSD at the first coupled period (T_{1m}), a slightly conservative value of the PDD can be determined for a given displacement response spectrum (e.g. AS 1170.4):

$$PDD_{flexible} = 1.6 \times RSD_{(T=T_{1M})} \tag{15}$$

5 CASE STUDY

Using the initial parametric study, some critical parameter values that govern the PDD were chosen for the two building types as case studies to evaluate the PDD using the different methods. The 2-storey structure for building Type 1 has the dimensions shown in Figure 1, giving a 2B/D of 2. A 2B/D of 1.4 (dimensions 29.4 x 21 m^2) and 5-storeys were used to represent the case study building for Type 2. The C-shaped walls were designed with requirements as per AS 3600 (Standards Australia, 2009) and for a 500-year return period event in Melbourne (or Sydney) using the AS 1170.4 (Standards Australia, 2007). A pushover analysis was used to calculate the force-displacement (F-Δ) relationship of the building, and for the different performance levels of cracking, yielding, serviceability, damage control and collapse prevention. These performance levels (for unconfined concrete) have the corresponding strains and drift limits given in Table 3. More on the process of the pushover analysis and definitions of the performance levels used can be found in Hoult et al. (2014) and Hoult et al. (2015). The walls are assessed as to whether they will remain elastic for a 500-year return period on soil class D using the acceleration-displacement response spectra format (and initially assuming no torsional effects). The stiffness of the building is then found from the pushover analysis ($K=F_{yield}/\Delta_{yield}$).

Table 3. Strain limits that determine the structure performance state for unconfined concrete

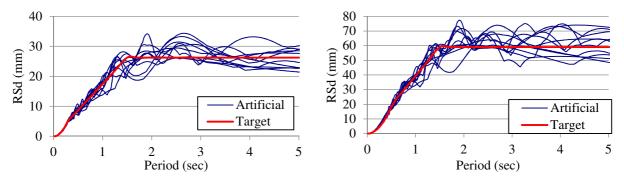| Structure Performance Limit State (Unconfined Concrete)	Concrete Strain (ε_c)	Steel Strain (ε_s)	Drift Limits (%)
Serviceability	0.0010	0.005	0.5
Damage Control	0.0015	0.010	1.5
Collapse Prevention	0.0030	0.015	-

Several methods are used to determine the PDD of the flexible side of the building; the time-history analysis (THA) of displacement response for a SDOF representation of the building is used to
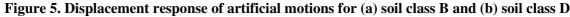
determine the PDD of 500-year return period ground motions for a site on rock (B) and soil class (D) given in Figure 5. SeismoArtif (SeismoSoft, 2013a) is used to compile the artificial ground motions as previous, while the spectra from AS 1170.4 (Standards Australia, 2007) were used as the targets. To complement and validate those results, non-linear THA were conducted using a state-of-the-art finite element modelling program SeismoStruct (SeismoSoft, 2013b). It is worth noting that although a non-linear THA is used in SeismoStruct, the ground motions used are such that the walls were found to remain elastic (as indicated in the initial pushover analysis). The estimated PDD from the T_{1m} (Equation 14) of the buildings is also used to ultimately compare with the THA methods.

The SeismoStruct results correlated strongly with the time-history analysis method (Equation 1-13) as illustrated for one of the ground motions for soil class B with building type 1 (Figure 6a) and soil class D with building type 2 (Figure 6b). The results for the PDD experienced at the flexible edge of the building for all ground motions using the time-history analysis for displacement response and SeismoStruct are given in Table 4.

It is likely that the buildings remained elastic due to the large period shift; there is an increase of the period from what was initially calculated by AS 1170.4 (Standards Australia, 2007) compared to what is found and used in the analyses. This was particularly true for when T_{1m} dominated; the T_{1m} values were typically high and little acceleration (and hence, force) was experienced relative to that calculated using the code method.

Using Equation 14, the T_{1m} was estimated for the two buildings and the corresponding PDD of the flexible edge was estimated using Equation 15 and the spectra from AS 1170.4 (Standards Australia, 2007). The results of the simplified estimation of the PDD are given in Table 5. The ratios of the estimated PDD (Equations 14-15) to the values obtained through the time-history analysis method (Equations 1-13) are also given in Table 5. This shows that the simplified expression for estimating PDD can give a reasonable and slightly conservative value of the expected displacement demand at the flexible edge of a building that is configured similarly to that given in Figures 1 or 2. It should be noted that although the buildings do not share similar T_{1m} values, the PDDs estimated in Table 5 are similar for both building types and for the different soil classes because the displacement response (demand) used has a "cut-off" RSD_{max}; the AS1170.4 has a corner period of 1.5s, and the displacement demand is constant for periods higher than this.

Figure 5. Displacement response of artificial motions for (a) soil class B and (b) soil class D
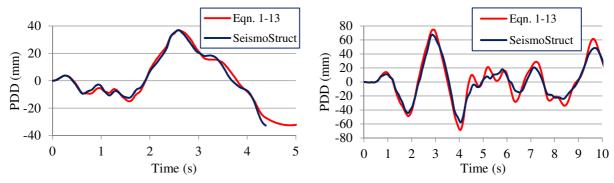

Figure 6. Time-history analysis and SeismoStruct results for (a) building type 1 with soil class B motion and (b) building type 2 with soil class D motion

Table 4. Comparison of PDD results (given in millimetres) from the time-history analysis (THA) method and SeismoStruct (SS) for building types 1 and 2 and range of ground motions (GM)

Type	Method	Soil	GM 1	GM 2	GM 3	GM 4	GM 5	GM 6	GM 7	GM 8	GM 9	GM 10	AVG
1	Eqn. 1-13	B	29	30	26	38	33	36	30	37	41	42	**34**
	Seismo-Struct	B	29	27	26	38	33	36	30	43	42	46	**35**
1	Eqn. 1-13	D	62	71	84	68	80	95	95	75	83	95	**81**
	Seismo-Struct	D	63	75	85	68	87	92	84	78	105	97	**83**
2	Eqn. 1-13	B	39	42	33	47	36	41	34	42	45	38	**40**
	Seismo-Struct	B	35	34	32	39	30	37	31	39	40	35	**35**
2	Eqn. 1-13	D	85	75	91	78	93	75	80	96	91	89	**85**
	Seismo-Struct	D	77	72	82	70	87	68	70	87	77	81	**77**

Table 5. Results of the estimated PDD for two building case study types using the simplified expression

Type	Soil	T_{1m} (s)	Estimated PDD (mm)		Ratio of **1** to **2**
			1 (Eqn. 14-15)	**2** (Eqn. 1-13)	
1	B	5.36	42	34	1.23
	D	5.36	94	81	1.17
2	B	2.36	42	40	1.06
	D	2.36	94	85	1.11

6 CONLCUSION

Low-rise single-core buildings, which are typically found in regions of low-to-moderate seismic regions, have the potential to cause large torsional displacements due to the asymmetry of the in-plan stiffness of structural elements. A simplified expression that estimates the PDD of the flexible edge of the building was found to give reasonable results in comparison to the more rigorous and time consuming methods. This simplified approach can be applied to a displacement-based design or assessment for buildings with similar configurations. This is particularly important for non-structural and structural elements, such as columns, that may be at locations that correspond with the flexible and crucial edge of the building. The ground motions selected for this research were such that the walls remained elastic. It is unclear if this simplified method will hold for when the walls reach and exceed yield, although the initial research and time-history analysis method from Lumantarna *et al.* (2013) indicates that it should hold for ductility's up to 2. Further research is being conducted at the University of Melbourne assessing the performance of the frames of such buildings with torsional displacement contributions.

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